

# AN ANALOGUE STUDY OF HEAT TRANSFER THROUGH PERIODICALLY CONTACTING SURFACES

J. R. HOWARD and A. E. SUTTON

Dept. of Mechanical Engineering and Dept. of Mathematics, University of Aston, Birmingham

(Received 16 October 1968 and in revised form 25 April 1969)

**Abstract**—An analogue computer was used to determine the effect of frequency and duration of contact per cycle on heat transfer through surfaces which are meeting and separating according to a regular cycle. The surfaces were of identical materials and perfect thermal contact and separation are assumed.

The results show that at high frequencies, the loss of heat transfer rate arising from the interruption of heat flow due to separation of the surfaces, is small and less dependent on duration of contact per cycle than at low frequencies. The relationship between loss of heat transfer rate, frequency and duration of contact is shown by a single curve.

## NOMENCLATURE

$C$ ,	distance from contact plane to point B (Fig. 1);		mean temperature difference from steady-state permanent contact condition;
$h$ ,	thermal conductance at contact plane;	$\Delta T_{1s}, \Delta T_{2s}, (T_A - T_{1s})$ and $(T_A - T_{2s})$ see Fig. 4(b);	
$H$ ,	distance from point A to contact plane (Fig. 1);	$x$ ,	distance;
$K_c$ ,	thermal conductivity of hotter member;	$\phi$ ,	heat flux — transfer rate per unit area;
$K_H$ ,	thermal conductivity of colder member;	$\alpha$ ,	thermal diffusivity;
$l l_b$ ,	length;	$\tau_c$ ,	time surfaces are in contact;
$L$ ,	dimensionless loss of heat transfer (equation 12);	$\tau_o$ ,	time surfaces are separated;
$t$ ,	time;	$f$ ,	frequency;
$T$ ,	temperature;	$\delta$ ,	depth below surface at which temperature fluctuation is negligible;
$T_c$ ,	temperature at any point in colder member;	$F(y), g(y)$ ,	function of $y$ .
$T_H$ ,	temperature at any point in hotter member;		
$T_o$ ,	temperature at contact plane with perfect thermal contact;		
$T_{oc}$ ,	temperature at contact plane on colder member;		
$T_{oH}$ ,	temperature at contact plane on hotter member;		
$\Delta T, \overline{\Delta T}$ ,	respectively, instantaneous and		

## 1. INTRODUCTION

THIS work is concerned with heat transfer through two surfaces which are undergoing a continuous, regular cycle of contact and separation. Practical examples of this, include that part of the heat transfer from the exhaust valve of an internal combustion engine which travels via the seating. While the valve is closed, the valve head is in contact with the valve seat

in the cylinder head and heat flows from the valve through the contacting surfaces. When the valve opens and the contact surfaces are separated, heat transfer is severely curtailed. Other examples are the heat transfer between work-piece and die in repetitive hot metal deformation processes and between soldering iron and workpieces.

In such cases, the heat transfer will depend upon the frequency and duration of contact, the overall temperature difference, thermal contact resistance at the contact surfaces, and the thermal properties of the materials in contact. A great deal of work has been done on thermal contact resistances of surfaces which are permanently in contact. References [1-3] contain valuable sources of information on both steady state and transient heat transfer.

However, in this present report, one-dimensional heat flow only is considered, with the simplifying assumptions of perfect thermal contact at the surfaces (i.e. no thermal contact resistance) and perfect thermal separation when the surfaces were not in contact.

The report describes an investigation using an analogue computer and forms part of a wider study by one of the authors (JRH). Current experimental work and further computer investigation now in progress will be reported later.

## 2. STATEMENT OF PROBLEM

### 2.1 General

Consider two bars of material AH and CB with their axes in line as shown in Fig. 1(a), and with one end  $H$  of one bar touching one end  $C$  of the other. If there were a steady, one-dimensional flow of heat along the axes of the bars i.e. no radial heat loss, then the temperature distribution would be as shown by line  $A, T_0, B$  in Fig. 1(b).

If then the ends of the bars were separated by a small distance and the temperature at  $A$  and  $B$  were to remain unchanged, the steady heat flow through the system would be very

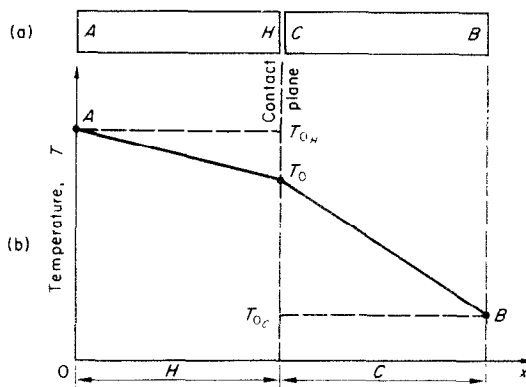


FIG. 1.

greatly reduced. Thus the temperature distribution would be given by  $A, T_0, T_{0C}$  and  $B$ . Clearly, under intermittent contact conditions, the temperature distribution will be between these two extremes, with the temperature near the contacting surfaces varying with time. Intermittent contact conditions will now be considered.

### 2.2 Assumptions

(i) The two bars are of equal cross-sectional area and have identical thermal properties.

(ii) When the two faces are brought together, perfect thermal contact is made. Under these circumstances, the temperature at the contact plane will change instantaneously [4] to the mean value of the two surface temperatures which existed just before contact was made.

(iii) When the surfaces are separated by a very small distance, the heat transfer rate is very small compared with that when the surfaces are in perfect thermal contact. For simplicity it is therefore assumed that when the surfaces are separated no heat transfer occurs.

(iv) Temperatures  $T_A$  and  $T_B$  are known. In our case we stipulate that they are at a fixed value for all time. This enables the temperature gradient at  $A$  and  $B$  to be determined. Instantaneous temperature distributions are shown in Fig. 2 for two cases; one with the surfaces in contact (line  $AbT_0dB$ ) the other when the sur-

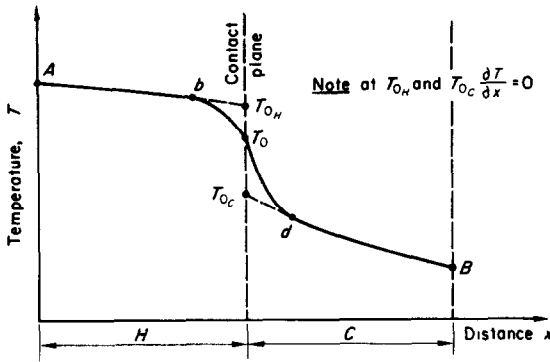


FIG. 2.

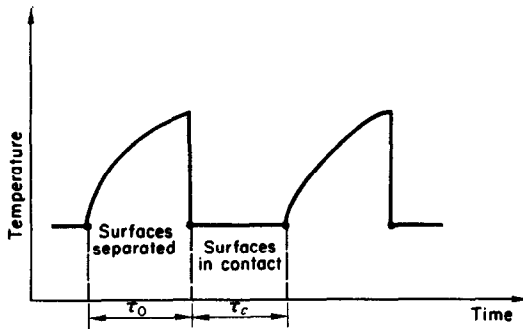


FIG. 3. Temperature/time relation at contact plane.

faces are separated (line  $AbT_{0H}T_{0c}dB$ ). Figure 3 shows the temperature-time relation at the contact plane of the hotter member.

2.3 Basic equation

Referring to Fig. 2 the heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} \right) \quad (1)$$

applies on either side of the contact plane, at all times.

Since the contact plane represents a discontinuity, equation (1) must be solved in four parts, because (a) the two members may be different materials, (although in our case identical materials are assumed) and (b) because of the two separate time periods; viz. surfaces in contact and when they are separated. Using the nomenclature given in the List of Symbols

and referring to Fig. 2 the following boundary conditions arise.

2.4 Boundary conditions

1.  $T_A$  and  $T_B$  fixed at all times.
2. At  $x = H$  and  $0 < t < \tau_c$  (i.e. during contact period)

$$T_{0H} = T_{0c}$$

[Note that this is only true when assumption (1) Section 2.2 is made].

3. At  $x = H$  and  $\tau_c < t < (\tau_c + \tau_0)$

$$k_H \left( \frac{\partial T_H}{\partial x} \right) = k_c \left( \frac{\partial T_c}{\partial x} \right) = 0.$$

2.5 Initial condition

$$T(x)_{t=0} = T_A - \left( \frac{T_A - T_B}{H + C} \right) x,$$

i.e. “steady state, surfaces permanently in contact” temperature distribution, chosen in order to reach the quasi-steady state rapidly.

2.6 Heat flow

The time-average heat flux is given by

$$\phi = -k_H \left( \frac{\partial T_H}{\partial x} \right)_A = -k_c \left( \frac{\partial T_c}{\partial x} \right)_B \quad (2)$$

3. ANALOGUE

3.1 The model

For convenience it was assumed that the two members were of identical material so that the temperature distribution would be symmetrical about the contact plane. The heat flux will be unaffected by the location of the plane of contact, so long as the zone in its immediate region, where the temperature is fluctuating, does not encroach on to the ends where the temperature is fixed (see Appendix A). Only the hotter member was therefore considered and it was divided into finite elements shown by the table of distances in Fig. 4(a). The steady state temperature distribution, with the end permanently in contact with the colder

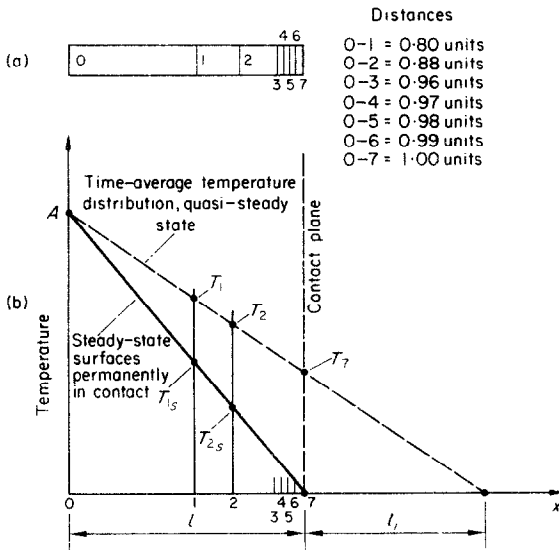


FIG. 4.

member is also shown in Fig. 4(b) by the line A7. The following material properties were assumed.

- Specific heat  $C = 460 \frac{\text{J}}{\text{kg degC}}$
- Density  $\rho = 7550 \frac{\text{kg}}{\text{m}^3}$
- Diffusivity  $\alpha = \frac{k}{\rho C} = 5 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$
- Thermal conductivity  $k = 17.4 \frac{\text{W}}{\text{m degC}}$
- Specimen length 0-7 = 0.04 m.

3.2 Dimensional analysis

Referring to the model Fig. 4 the heat transfer rate per unit cross-sectional area  $\phi$ , depends on the temperature difference ( $T_A - T_7$ ) between the ends, the overall length  $l$ , thermal conductivity  $K$ , thermal diffusivity  $\alpha$ , frequency of contact  $f$  and duration of contact per cycle ( $f\tau_c$ ).

Boundary condition 1, Section 2.4, implies that ends  $A$  and  $B$  of the two bars, Figs. 1 and

2, are in perfect thermal contact with heat reservoirs at temperatures  $T_A$  and  $T_B$  respectively, of infinite heat capacity and made of a substance whose thermal conductivity is infinite.

If instead the bars were in perfect thermal contact at  $A$  and  $B$  with a system of finite properties then the temperature at  $A$  and  $B$  would fluctuate due to the periodic interruption of the heat flow at the contact plane. However, if the length of the bars is sufficiently large then the amplitude of temperature fluctuations at  $A$  and  $B$  would be very small once a "quasi-steady" state had been reached. If length  $l$  equals  $\delta$  in equation (3) below, [5, 6],

$$\delta = 1.6 \sqrt{\frac{l\alpha}{f}} \tag{3}$$

and the temperature at the contact plane varies sinusoidally, the amplitude of temperature fluctuation at depth  $\delta$  from the contact plane is only 0.66 per cent of that at the contact plane.

To test the accuracy of the analogue simulation, the amplitude of the temperature at stations 2 and 7 in the rod, Fig. 4(a), were compared. The ratio of these two amplitudes agreed closely with that computed from the exact solution of the heat diffusion equation (1) for a semi-infinite solid whose surface temperature varies periodically with time [6].

Furthermore, once  $l$  exceeds  $\delta$  and providing that the time-average heat flux is unchanged, an increase in  $l$  will only introduce an additional series thermal resistance into the system. Thus the thermal resistances of the system consists of two independent thermal resistances in series,  $R_c$  and  $R_i$ ,  $R_c$  being the resistance under continuous contact conditions due to the length  $l$  of the conducting material (of unit cross-sectional area) and  $R_i$  being due to the periodic interruption of the heat flow.

$R_i$  may also be represented by a length,  $l_i$ , of the same conducting material of unit cross-sectional area.

It is desirable therefore to choose dimensionless groups which reflect the independence of

$R_s$  and  $R_b$  isolating  $l$  in one of them. Note:  $R_i = F[f, (f\tau_c)]$ .

Notice that if the contact plane is situated at a distance less than  $\delta$  from the extreme ends  $A$  and  $B$  of the two bars (a case we have not considered), then the loss of heat flow is reduced, until when the contact plane is at the end, the loss of heat flow is half that when the contact plane is located at a distance greater than  $\delta$  from the ends.

Referring again to Fig. 4(b) which shows the quasi-steady state time-average temperature distribution when the surfaces are meeting and parting regularly, together with the distribution under steady state surfaces permanently in contact condition, it will be seen that the loss of heat flux due to periodic interruption of the heat flow is given by

$$\phi_s - \phi = \frac{k(T_1 - T_{1s})}{(x_1 - x_0)} = \frac{k(T_2 - T_{2s})}{x_2 - x_0} = \frac{k(T_7 - T_{7s})}{(x_7 - x_0)}. \quad (4)$$

This may be expressed non-dimensionally as

$$L = \frac{\phi_s - \phi}{\phi_s} = \frac{T_1 - T_{1s}}{T_A - T_{1s}} = \frac{T_7 - T_{7s}}{T_A - T_{7s}} = \frac{l_i}{l + l_i}, \quad (5)$$

whence

$$l_i = \frac{lL}{1 - L}. \quad (6)$$

The relationship between the dimensionless parameters can be expressed in the form

$$\left(\frac{fl_i^2}{\alpha}\right) = g \left[ \left(\frac{fl^2}{\alpha}\right), (f\tau_c) \right]. \quad (7)$$

But using equation (3), when  $(fl^2/\alpha) > 2.56\pi$ ,  $l_i$  is independent of  $l$  and hence  $(fl_i^2/\alpha)$  is independent of  $(fl^2/\alpha)$

Hence

$$\left(\frac{fl_i^2}{\alpha}\right) = g(f\tau_c) \text{ only.} \quad (8)$$

Thus, the number of significant dimensionless groups involved is two instead of three, with the advantage of saving a complete dimension of computation without loss of generality.

In our experiment the values of  $(fl^2/\alpha)$  ranged from 4.64 to 3210.

### 3.3 Finite-difference equations

Equation (1) was written in finite-difference form.

At any position on the model the temperature difference  $T$  between the "steady-state surfaces permanently in contact" condition and the temperature when the surfaces are meeting and parting regularly (and quasi-steady state is reached) is described by the equations below. Thus, referring to Fig. 4(b),  $\Delta T_1 = T_1 - T_{1s}$ , etc. i.e.  $\Delta T_1$  is the difference between the actual temperature in the quasi-steady state and the temperature when the steady state surfaces permanently in contact is attained. Putting  $D \equiv (d/dt)$

$$D(\Delta T_1) = 0.089 \Delta T_2 - 0.098 \Delta T_1 + 0.0089 \Delta T_0 \quad (9)$$

in which  $\Delta T_0$  is zero since our boundary condition at position 0 is that the temperature remains fixed for all cases.

$$D(\Delta T_2) = 0.488 \Delta T_3 - 0.976 \Delta T_2 + 0.488 \Delta T_1 \quad (10)$$

$$D(\Delta T_3) = 6.95 \Delta T_4 - 7.82 \Delta T_3 + 0.87 \Delta T_2 \quad (11)$$

$$D(\Delta T_4) = 31.25 \Delta T_5 - 62.5 \Delta T_4 + 31.25 \Delta T_3 \quad (12)$$

$$D(\Delta T_5) = 31.25 \Delta T_6 - 62.5 \Delta T_5 + 31.25 \Delta T_4 \quad (13)$$

$$D(\Delta T_6) = 31.25 \Delta T_7 - 62.5 \Delta T_6 + 31.25 \Delta T_5. \quad (14)$$

When surfaces are in contact

$$\Delta T_7 = 0. \quad (15)$$

When surfaces are separated

$$D(\Delta T_7) = 62.5 \Delta T_6 - 62.5 \Delta T_7 + 62.5. \quad (16)$$

### 3.4 Circuitry

Figure 5 shows the circuit diagram employed on a PACE analogue computer.

Meeting and parting of the surfaces was simulated by the closing and opening of a switch whose frequency of operation and duration of closure could be varied. The switch was connected across amplifier 4 as shown in Fig. 5.

Outputs  $\Delta T_1$  and  $\Delta T_2$  were measured with a digital voltmeter. At low frequencies of contact however, these outputs fluctuated cyclically and they were then measured with an U.V. recorder and time mean values determined.

Output  $\Delta T_7$  was measured with an U.V. recorder at all times so that frequency and duration of contact could be determined (A typical trace of  $\Delta T_7$  is shown in Fig. 6.)

## 4. PROCEDURE

### 4.1 Temperature distribution

The computer was switched on with the variable-frequency switch permanently open. When steady conditions were reached, outputs  $\Delta T_1, \Delta T_2, \dots, \Delta T_7$  were measured with the digital voltmeter. Thus the temperature distribution under "steady-state surfaces permanently in contact" condition was obtained. At this condition, see Fig. 4(b),

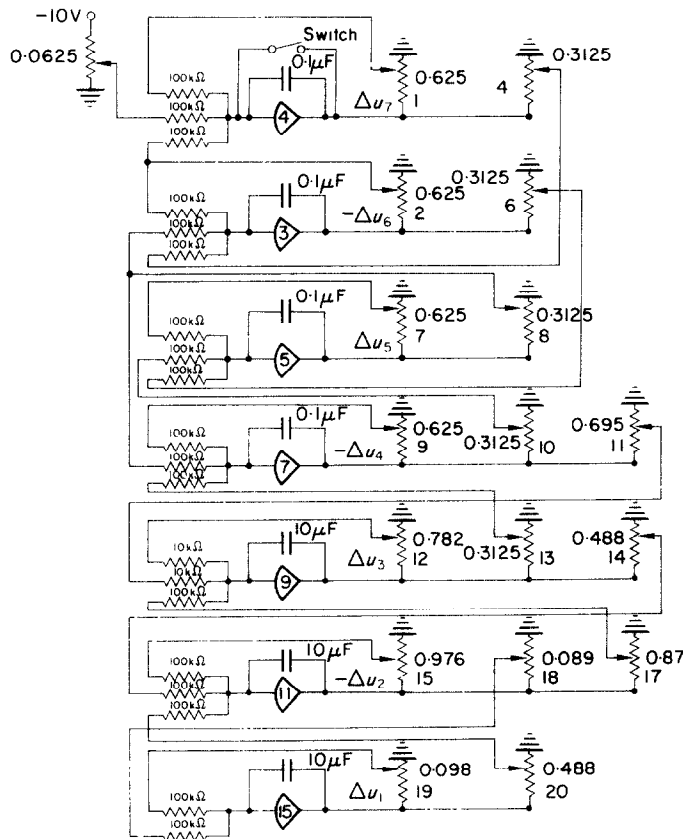


FIG. 5.



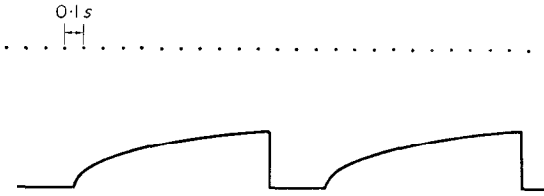


FIG. 6. Typical trace of  $\Delta T$ .

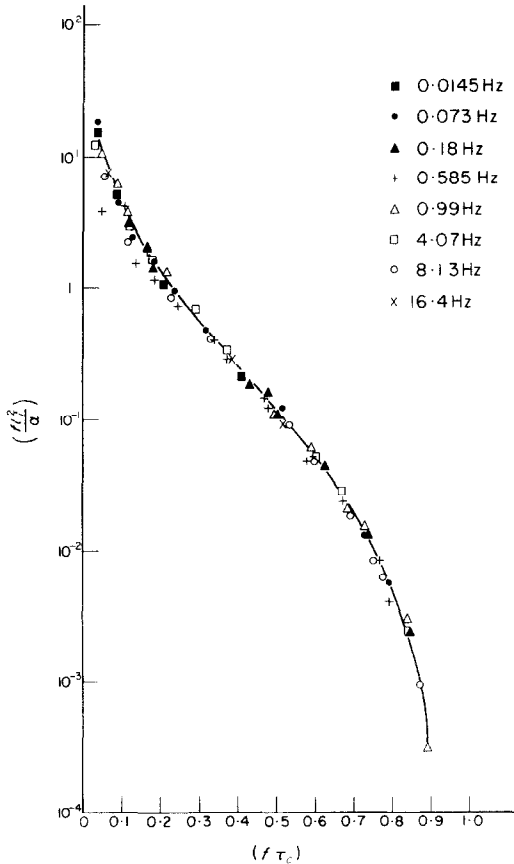


FIG. 7.

whence  $L$  tends to a value given by

$$L = \left(\frac{fl^2}{\alpha}\right)^{-\frac{1}{2}} \{g(f\tau_c)\}^{\frac{1}{2}} \quad (21)$$

If the frequency of contact with a given system (of fixed  $l$  and  $\alpha$ ) is varied while  $(f\tau_c)$  is maintained constant, then from equations (6)

and (8), since  $L$  cannot be negative or exceed unity,

$$\frac{L}{1-L} = + (p/f)^{\frac{1}{2}} \quad (22)$$

where 
$$p = \frac{\alpha \cdot g(f\tau_c)}{l^2} \quad (23)$$

giving 
$$L = \frac{(p/f)^{\frac{1}{2}}}{1 + (p/f)^{\frac{1}{2}}} \quad (24)$$

Thus, as  $f$  increases so  $L$  falls, and at sufficiently large values of  $(f\tau_c)$  i.e. small  $p$ , loss  $L$  of heat flow brought about by periodic interruption of heat flow will be small.

It should be emphasised that the analogue does not solve the partial differential equation (1) but only the finite-difference approximations to it, equations (9)–(16); which includes the boundary condition that the temperature at the hottest end of the bar is fixed i.e.  $\Delta T_0 = 0$ , equation (9). In practical cases the amplitude of temperature fluctuation within the rod will decay exponentially [6] with distance from the contact planes. At the lowest frequency investigated, 0.0145 Hz the amplitude at the hottest end of the bar would only amount to 2.2 per cent of that at the contact plane, thus approximating to the boundary condition closely.

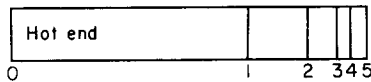
Since a non-uniform division of the model Fig. 4(a) was employed it is difficult to estimate the error due to the finite-difference approximation. However, comparison of these results with some obtained using the division shown in Fig. 8 shows them to be in close agreement.

It is emphasised that the data obtained is applicable only to the case where both hot and cold members are of identical material and perfect contact and separation occurs at the plane of contact. Cases where the members are of different materials and where thermal contact resistance is present are being studied currently and will be dealt with in a later report.

Clearly the results suggest that in practical cases, at sufficiently high values of  $(f\tau_c)$  and



Not to scale



$$0-1 = 0.8 \text{ units}$$

$$0-2 = 0.96 \text{ units}$$

$$0-3 = 0.992 \text{ units}$$

$$0-4 = 0.9984 \text{ units}$$

$$0-5 = 1.0 \text{ units}$$

FIG. 8. Earlier division of model.

frequency thermal contact resistance at the contact plane will exert a more significant effect on heat flow than the periodic interruption at the contact plane.

#### ACKNOWLEDGEMENTS

The thanks of the authors are due to Dr. W. P. Mansfield, Director of Research, British Internal Combustion Engine Research Institute for suggesting the wider study currently being carried out by J. R. Howard and to Professor A. J. Ede, Head of the Department of Mechanical Engineering, University of Aston in Birmingham for his encouragement, help and, valuable criticism.

Finally we must express our gratitude to Dr. J. Barber of the Department of Mechanical Engineering, University of Newcastle upon Tyne for most painstaking and constructive criticism which led to the presentation of results in the present form.

#### REFERENCES

1. H. Y. WONG, A Survey of the thermal conductance of metallic contacts. Ministry of Technology Aeronautical Research Council Current Papers, C.P. No. 973. H.M.S.O. (1968).
2. H. ATKINS, Bibliography on thermal metallic contact conductance. N.A.S.A. Technical Memorandum TM X-53227. (15 April, 1965).
3. H. Y. WONG, Ph.D. Thesis No. 3011, University of Glasgow, Department of Aeronautics and Fluid Mechanics (1968).
4. H. GROBER, S. ERK and U. GRIGULL, *Fundamentals of Heat*, 3rd edn., pp. 133-136, equation 6.14. McGraw-Hill.
5. E. R. G. ECKERT and R. M. DRAKE, *Heat and Mass Transfer*, p. 106 Ex 4-7. McGraw-Hill, New York (1959).
6. E. R. G. ECKERT and R. M. DRAKE, *Heat and Mass Transfer* p. 102. McGraw-Hill, New York (1959).

#### APPENDIX

Consider Fig. 9, which shows the temperature distribution in the case where the hot and cold members are made of identical material of thermal conductivity  $k$ ,

- (i) their lengths are not equal
- (ii) a thermal contact resistance  $(1/h)$  exists at the plane of contact between hot and cold members. This introduces a discontinuity at the plane of contact.

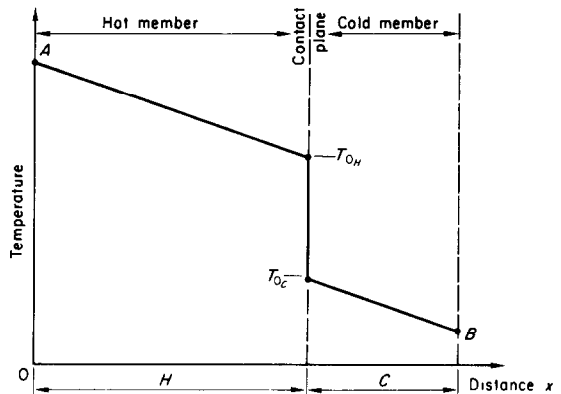


FIG. 9

- (iii) the hot and cold members are permanently in contact.

Clearly the heat flux  $\phi$  is given by

$$\phi = \frac{k(T_A - T_{0H})}{H} = \frac{k(T_{0C} - T_B)}{C} = h(T_{0H} - T_{0C}), \quad (25)$$

which on eliminating  $T_{0H}$  and  $T_{0C}$  gives

$$\phi = \left\{ \frac{T_A - T_B}{(1/k)(H + C) + (1/h)} \right\}. \quad (26)$$

Thus, for given end temperatures  $T_A$  and  $T_B$ , overall length of the system  $(H + C)$  and contact resistance  $(1/h)$ , the heat flux  $\phi$  is independent of its position between the ends  $A$  and  $B$ .

Consider now the case where there is perfect thermal contact between the hot and cold

members when they are brought together and no heat transfer between the members when they are separated.

Figure 10 shows the temperature distribution when the members are meeting and parting at the plane of contact at a given frequency and ratio of contact time: periodic time ( $f\tau_c$ ) and when quasi-steady conditions are reached.

The temperature in the immediate region of the plane of contact will vary with time but at some depth  $\delta$  below the surface, the fluctuation of temperature will be negligible [5]. The shaded areas in Fig. 10 are bounded by the maximum and minimum temperature reached during a cycle of contact and separation of the two members. The time average temperature distribution however is given by line  $ADT_{O_{Hav}}T_{O_{Cav}}EB$ . This is of the same form as in Fig. 10 and hence again the heat flux is independent of the position of the contact plane between the ends  $A$  and  $B$  provided of course, that it is not positioned so close to  $A$  or  $B$  that the temperature at  $A$  and  $B$  fluctuates,

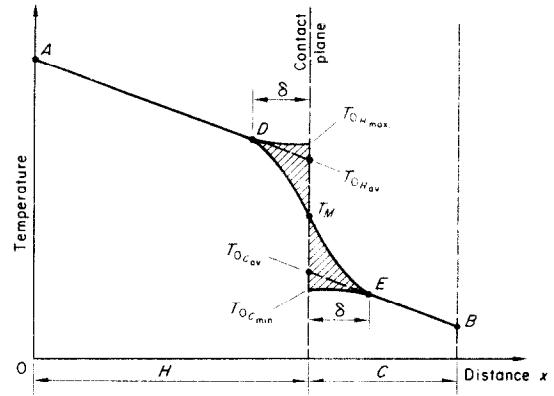


FIG. 10

thus invalidating our boundary condition of fixed temperatures at  $A$  and  $B$ . However, if we stipulated that the temperature at  $A$  and  $B$  were allowed to fluctuate but with a fixed time-average value, the heat flux  $\phi$  would remain independent of the position of the contact plane over the entire distance between  $A$  and  $B$ .

#### ETUDE ANALOGIQUE DU TRANSPORT DE CHALEUR À TRAVERS DES SURFACES PÉRIODIQUEMENT EN CONTACT

**Résumé**—Un calculateur analogique a été employé pour déterminer l'effet de la fréquence et de la durée de contact par cycle sur le transport de chaleur à travers des surfaces qui se rejoignent et se séparent selon un cycle régulier. Les surfaces étaient constituées par des matériaux identiques et l'on a supposé que le contact et la séparation thermique étaient parfaits.

Les résultats montrent qu'à des fréquences élevées, la perte de vitesse de transfert de chaleur provenant de l'interruption du flux de chaleur, due à la séparation des surfaces, est faible et dépendant moins de la durée de contact par cycle qu'aux basses fréquences. La relation entre la diminution de la vitesse de transfert de chaleur, la fréquence et la durée de contact est montrée par une courbe unique.

#### EINE ANALOGIE-UNTERSUCHUNG DES WÄRMEDURCHGANGS DURCH PERIODISCH SICH BERÜHRENDE FLÄCHEN

**Zusammenfassung**—Mit Hilfe eines Analogrechners wurde der Einfluss von Frequenz und Dauer des Kontakts pro Periode auf den Wärmeübergang durch Flächen untersucht, die in regelmässiger Folge zusammengeführt und wieder getrennt werden.

Die Flächen waren aus gleichem Material, es wurde vollständiger thermischer Kontakt und vollständige Trennung vorausgesetzt.

Die Ergebnisse zeigen, dass bei hohen Frequenzen die Abnahme des Wärmeübergangs bei der Unterbrechung des Wärmestroms durch Trennung der Flächen klein bleibt und die Kontaktdauer von geringerem Einfluss ist als bei kleinen Frequenzen. Der Zusammenhang zwischen Abnahme des Wärmeübergangs, Frequenz und Kontaktdauer wird durch eine einzige Kurve dargestellt.

#### АНАЛОГОВОЕ ИССЛЕДОВАНИЕ ТЕПЛООБМЕНА ЧЕРЕЗ ПЕРИОДИЧЕСКИ КОНТАРТНЫЕ ПОВЕРХНОСТИ

**Аннотация**—С помощью аналоговой вычислительной машины определялось влияние частоты и длительности контакта в течение цикла на перенос тепла через встречающиеся и расходящиеся в соответствии с редулярным циклом поверхности. Поверхности

изготовлены из идентичного материала и принято, что тепловой контакт и разделение идеальные.

Результаты показывают, что при высоких частотах уменьшение теплового потока, возникающее из-за разделения поверхностей, невелико и меньше зависит от длительности контакта в течение цикла, чем при низких частотах. Соотношение между уменьшением теплового потока, частотой и длительностью контакта представлено одной кривой.